**1. Derive the joint distribution π(y, N, β).**

Considering N and β are independent, thus

Consequently,

**2. Derive full conditional distribution π (N|β, y) and π (β |N, y).**

To get π (β |N, y), we just get rid of all the terms irrelevant to β, which is

Normalizing it, we can get

For π(N|β, y), we get rid of all the terms irrelevant to N, which is

Normalizing it, we can get

**3. Use these to sample (using Gibbs sampling) from the joint posterior (β, N | y) using a starting value of (β(1), N(1)))=(0.05,50).**

The sampling process is recorded in the R code attached.

**4. Show trace plots for β and N.**

Trace Plot for N:



Trace Plot to β:



**5. Show the 2D trace plot for the first 10 draws of the Gibbs sampler, (β (1:10), N(1:10)). Show both the points and the connecting lines.**



**6. Give the central 90% posterior credible interval for β, accurate to (and rounded to) the nearest 1% for both upper and lower limits.**

The central 90% posterior credible interval for β is (0.64, 0.99). Calculation is in attached code.

**7. What is the probability that exactly 20 people were polled? Base your answer on at least 10,000 draws (post-burn-in), and round to the nearest one tenth of 1%.**

The probability that exactly 20 people were polled is achieved when N=20, which is 36.4%, after doing 100 burn-in steps and 10000 post-burn-in steps.